Product differentiation

- Consumers have preferences over product characteristics
- Vertical differentiation vs horizontal differentiation
- Address model: consumers are distributed on a unidimensional product space (1 characteristics).
- The utility of a consumer located at $x$ who purchases a good «located» at $x_i$ is
  $$V - t |x - x_i| - p_i$$

Hotelling model

- First stage: firms choose product differentiation (location)
- Second stage: given location, firms compete in price.
- Solve backward to find the SPNE
**Hotelling model: second stage**

with firms located at the two extremes

- The indifferent consumer’s address is such that
  \[ p_0 + t\bar{x} = p_1 + t(1 - \bar{x}) \]
- All consumers to the left of \( x \) buy from firm “0”, those on the right go to firm “1”:
  \[ d_0(p_0, p_1) = \bar{x} = \frac{1}{2} - \frac{p_0 - p_1}{2t} \]

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**Hotelling model: second stage**

with firms located at the two extremes

- Find the reaction function for firm “0”
  \[ \Pi_0 = (p_0 - c)d_0 = (p_0 - c)\left(\frac{1}{2} - \frac{p_0 - p_1}{2t}\right) \]
  \[ \frac{d\Pi_0}{dp_0} = 0 \rightarrow p_0 = \frac{p_1 + t + c}{2} \]
- And for firm 1
  \[ p_1 = \frac{p_0 + t + c}{2} \]
- Solve to get the Nash equilibrium
  \[ p_0 = p_1 = t + c \]
### Graphically

\[ p^*_1 = \frac{p_2 + t + c}{2} \]
\[ p^*_2 = \frac{p_1 + t + c}{2} \]
\[ \therefore p^*_2 = t + c \]
\[ \therefore p^*_1 = t + c \]

### Hotelling model

- **Second stage**
  - **QUADRATIC COSTS**
  - \[ V - p_1 \cdot t(x^m - a)^2 = V - p_2 \cdot t(1 - b - x^m)^2 \]

\[
D^1(a, b, p_1, p_2) = x^m = a + \frac{1 - a - b}{2} + \frac{p_2 - p_1}{2t(1 - a - b)} \]
\[
D^2(a, b, p_1, p_2) = 1 - x^m = b + \frac{1 - a - b}{2} + \frac{p_1 - p_2}{2t(1 - a - b)} \]

\[
\Pi^1(a, b, p_1, p_2) = (p_1 - c)D^1(a, b, p_1, p_2), \]
\[
\Pi^2(a, b, p_1, p_2) = (p_2 - c)D^2(a, b, p_1, p_2) \]
**Hotelling model: second stage**

- Solving

\[
\frac{\partial \Pi^1}{\partial p_1} = p_2 - 2p_1 + c - t\left[a^2 - (1-b)^2\right] = 0 \quad \frac{\partial \Pi^2}{\partial p_2} = p_1 - 2p_2 + c - t\left[b^2 - (1-a)^2\right] = 0
\]

\[
p_1 = \frac{1}{2}\left\{p_2 + c - t\left[a^2 - (1-b)^2\right]\right\} \quad p_2 = \frac{1}{2}\left\{p_1 + c - t\left[b^2 - (1-a)^2\right]\right\}
\]

\[
p_1^*(a,b) = c + t(1-b-a)\left(1 + \frac{a-b}{3}\right)
\]

\[
p_2^*(a,b) = c + t(1-b-a)\left(1 + \frac{b-a}{3}\right)
\]

- I prezzi di entrambi calano con a: quando i due prodotti si avvicinano, diventano meno differenziati e la competizione di prezzo si fa più intensa!

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**Hotelling Model**

\[
\Pi^1[a,b, p_1^*(a,b), p_2^*(a,b)] = \left[ p_1^*(a,b) - c \right] D^1[a,b, p_1^*(a,b), p_2^*(a,b)]
\]

\[
\Pi^2[a,b, p_1^*(a,b), p_2^*(a,b)] = \left[ p_2^*(a,b) - c \right] D^2[a,b, p_1^*(a,b), p_2^*(a,b)]
\]

\[
\frac{\partial \Pi^1}{\partial a} + \frac{\partial \Pi^1}{\partial p_1} \frac{\partial p_1^*}{\partial a} + \frac{\partial \Pi^1}{\partial p_2} \frac{\partial p_2^*}{\partial a}
\]

**Demand (Hotelling) effect:** it is better go go closer to your rival so as to capture more demand → move towards the center

**Price (strategic) effect:** to reduce price competition it is better to move away from your rival and differentiate your products
Product differentiation

- When transportation costs are linear, the demand effect dominates → minimum differentiation
- When transportation costs are quadratic, the price effect dominates → maximum differentiation

Vertical differentiation